PRAYAS

JEE 2026

Mathematics

Basic Maths

Lecture - 10

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Topics *to be covered*

- **A** Logarithm and its Properties
- **B** Problem Practice

Homework Discussion

QUESTION

(Home Challenge-02)

Solve in real numbers the equation

$$\sqrt{x_1-1}+2\sqrt{x_2-4}+\cdots+n\sqrt{x_n-n^2}=\frac{1}{2}(x_1+x_2+\cdots+x_n) \text{ for } x_1, x_2, x_3 \ldots, x_{n-1}, x_n.$$

$$2\sqrt{x_1-1}+4\sqrt{x_2-4}+\cdots+2n\sqrt{x_n-n^2}=x_1+x_2+\cdots+x_n$$

$$\sqrt{x_1-1}^2-2\sqrt{x_1-1}+1+\sqrt{x_2-4}^2-4\sqrt{x_2-4}+4+\sqrt{x_3-9}^2-6\sqrt{x_n-9}+9+\cdots$$

$$\cdots+\sqrt{x_n-n^2}^2-2n\sqrt{x_n-n^2}+n^2=0$$

$$\Rightarrow\left(\sqrt{x_1-1}-1\right)^2+\left(\sqrt{x_2-4}-2\right)^2+\left(\sqrt{x_3-9}-3\right)^2+\cdots+\left(\sqrt{x_n-n^2}-n\right)^2=0$$

$$\sqrt{x_1-1}=1\Rightarrow x_1=2$$

$$\sqrt{x_2-4}=2\Rightarrow x_2=8$$

$$\sqrt{x_3-9}=3\Rightarrow x_3=18$$

$$\vdots$$

$$\sqrt{x_n-n^2}=n\Rightarrow x_n=2n^2$$

Aao Machaay Dhamaal
Deh Swaal pe Deh Swaal

★ $\log_b a = \dfrac{\log_c a}{\log_c b}$

$\log_b a = \dfrac{1}{\log_a b}$

$a^{\log_c b} = b^{\log_c a}$

D_3 : $\log_{a_1} a_2 \cdot \log_{a_2} a_3 \cdot \log_{a_3} a_4 \, ... \, \log_{a_{n-1}} a_n = \log_{a_1} a_n$

proof: $\log_{a_1} a_2 \cdot \log_{a_2} a_3 \; \log_{a_3} a_4 \; --- \; \log_{a_{n-1}} a_n$

$$= \frac{\log_{10} a_2}{\cancel{\log_{10} a_1}} \cdot \frac{\cancel{\log_{10} a_3}}{\cancel{\log_{10} a_2}} \cdot \frac{\cancel{\log_{10} a_4}}{\cancel{\log_{10} a_3}} --- \frac{\log_{10} a_n}{\cancel{\log_{10} a_{n-1}}}$$

$$= \frac{\log_{10} a_n}{\log_{10} a_1} = \log_{a_1} a_n = \underbrace{\text{RHS}}$$

Ex: $\log_2 3 \cdot \log_3 4 \cdot \log_4 5 \cdots \log_{31} 32 = ?$

$$\log_2 32 = \log_2 2^5 = 5\log_2 2 = 5.$$

D_4 : $\log_{a^y} m^x = \frac{x}{y}\log_a m$ where $a, m > 0,\ a \neq 1, x, y \in R, y \neq 0$

proof: $\log_{a^y} m^x = \dfrac{\log_a m^x}{\log_a a^y} = \dfrac{x \log_a m}{y \log_a a} = \dfrac{x \log_a m}{y} = \dfrac{x}{y}\log_a m$

Ex: $\log_{16} 32 = \log_{2^4} 2^5 = \frac{5}{4}\log_2 2 = 5/4$

Ex: $\log_{\sqrt[3]{9}} \sqrt[4]{3} = \log_{3^{2/3}} 3^{1/4} = \dfrac{1/4}{2/3}\log_3 3 = \frac{3}{8}$ Ans

$D_5 : \quad a^{\sqrt{\log_a b}} = b^{\sqrt{\log_b a}}$

$\underbrace{\text{LHS}}$ $\quad a^{\sqrt{\log_a b}} = a^{\frac{\log_a b}{\sqrt{\log_a b}}} = \left(a^{\log_a b}\right)^{\frac{1}{\sqrt{\log_a b}}}$ $\quad \left(\because a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}}\right)$

$\left(\because \sqrt{x} = \frac{x}{\sqrt{x}}\right)$ $x \in R^+$

$= b^{\sqrt{\log_b a}} = \underline{RHS}$

Ex: $2^{\sqrt{\log_2 3}} - 3^{\sqrt{\log_3 2}} = ?$

$3^{\sqrt{\log_3 2}} - 3^{\sqrt{\log_3 2}} = 0$

Kaam Ki Baat

Common base : $\log x = \log_{10} x$

Natural base : $\log_e x = \ln(x)$

(where $e \cong 2.718$ & $e^2 \cong 7.389$)

 # In Calculus : log x is considered as log with natural base.

$\ln 2 \cong 0.693$,
$\ln 3 \cong 1.098$,
$\ln 5 \cong 1.609$,
$\ln 7 \cong 1.945$,

$\log_e 10 \simeq 2.303$

$\log_{10} x = \dfrac{\ln x}{2.303}$

How??

* $\ln x + \ln y = \ln(xy)$
* $\ln x^n = n \ln x$

$\log_{10} x = \dfrac{\log_e x}{\log_e 10} = \dfrac{\ln x}{2.303}$

QUESTION

	Column 1			Column 2
(a)	$\frac{1}{9}^{\log_3 7}$	R	(P)	1/27
(b)	$2^{-\log_{1/2} 7}$	S	(Q)	1/2
(c)	$8^{-\frac{1}{\log_3 2}}$	P	(R)	1/49
(d)	$\left(\frac{1}{4}\right)^{\log_2 6}$		(S)	7
(e)	$3^{-\log_3 2}$		(T)	1/36

(d) $= 6^{\log_2 \frac{1}{4}} = 6^{-2} = \frac{1}{36}$

(e) $= \left(2^{\log_3 3}\right)^{-1} = 2^{-1} = 1/2$ (Q)

Interchange

(a) $\left(\frac{1}{9}\right)^{\log_3 7} = 7^{\log_3 \frac{1}{9}}$

$= 7^{-2} = \frac{1}{7^2} = \frac{1}{49}$

(b) $2^{-\log_{\frac{1}{2}} 7} = \left(2^{\log_{\frac{1}{2}} 7}\right)^{-1}$

$= \left(7^{\log_{\frac{1}{2}} 2}\right)^{-1}$

$= (7^{-1})^{-1} = 7^1 = 7$

(c) $8^{-\frac{1}{\log_3 2}} = 8^{-\log_2 3}$

$= \left(8^{\log_2 3}\right)^{-1} = \left(3^{\log_2 8}\right)^{-1}$

$= (3^3)^{-1} = 3^{-3} = 1/27$

QUESTION

$A = \log_{11}\left(11^{\log_{11} 1331}\right)$; $B = \log_{385} 5 + \log_{385} 11 + \log_{385} 7$

$C = \log_4\left(\log_2(\log_5 625)\right)$; $D = 10^{\log_{100} 16}$

Find the value of A × B ÷ C – D

$A = \log_{11} 1331 = 3$

$B = \log_{385}(5 \times 11 \times 7) = \log_{385} 385 = 1$

$C = \log_4(\log_2 4) = \log_4 2 = 1/2$

$D = 10^{\log_{100} 16} = 16^{\log_{100} 10} = 16^{\log_{10^2} 10} = 16^{\frac{1}{2}\log_{10} 10} = 16^{1/2} = \sqrt{16} = 4$

$A \times B \div C - D$

$3 \times 1 \div 1/2 - 4$

$3 \times 1 \times 2 - 4$

$= 6 - 4 = 2$ Ans

QUESTION

Find the value of $\frac{1}{\log_3 2}+\frac{2}{\log_9 4}-\frac{3}{\log_{27} 8}=$ $\log_2 3 + 2\log_4 9 - 3\log_8 27$

$= \log_2 3 + 2\cdot\log_{2^2} 3^2 - 3\log_{2^3} 3^3$

$= \log_2 3 + 2\cdot\frac{\cancel{2}}{\cancel{2}}\log_2 3 - 3\cdot\frac{\cancel{3}}{\cancel{3}}\log_2 3$

$= \log_2 3 + 2\log_2 3 - 3\log_2 3$

$= 0$

QUESTION

Find the value of $\log_2 10 - \log_8 125$.

$\log_2 10 - \log_{2^3} 5^3$

$\log_2 10 - \frac{3}{3}\log_2 5$

$\log_2 10 - \log_2 5$

$= \log_2 (10/5) = \log_2 2 = 1$

QUESTION

If $\log_2 3 \cdot \log_3 4 \cdot \log_4 5 \ldots\ldots \log_n (n+1) = 10$ find n.

$$\log_2 (n+1) = 10$$

$$n+1 = 2^{10}$$

$$n = 1023 \text{ Ans}$$

$$\log_{a_1} a_2 \cdot \log_{a_2} a_3 \cdot \log_{a_3} a_4 \,---\, \log_{a_{n-1}} a_n$$

$$\parallel$$

$$\log_{a_1} a_n$$

QUESTION

Simplify : $\dfrac{81^{\frac{1}{\log_5 9}}+3^{\frac{3}{\log_{\sqrt{6}} 3}}}{409}\cdot\left((\sqrt{7})^{\frac{2}{\log_{25} 7}}-(125)^{\log_{25} 6}\right)$

$$a^{m\log_c b}=b^{m\log_c a}$$

$$\left(a^{\log_c b}\right)^m$$

$$\left(b^{\log_c a}\right)^m$$

$$=b^{m\log_c a}$$

$$\frac{81^{\log_9 5}+3^{3\cdot\log_3 \sqrt{6}}}{409}\left(\sqrt{7}^{\,2\log_7 25}-125^{\log_{25} 6}\right)$$

$$=\frac{5^{\log_9 81}+\sqrt{6}^{\,3\log_3 3}}{409}\left(25^{2\log_7 \sqrt{7}}-6^{\log_{25} 125}\right)$$

$$=\frac{\left(5^2+\sqrt{6}^{\,3}\right)}{409}\left(25^{2\cdot 1/2}-6^{\log_{5^2} 5^3}\right)$$

$$=\frac{(25+6\sqrt{6})\cdot(25-6^{3/2})}{409}=\frac{(25+6\sqrt{6})(25-6\sqrt{6})}{409}=\frac{625-216}{409}=1$$

$$6^{3/2}=6^{1+1/2}$$

$$=6^1\cdot 6^{1/2}$$

$$=6\sqrt{6}.$$

QUESTION

Simplify : $7^{\log_3 5} + 3^{\log_5 7} - 5^{\log_3 7} - 7^{\log_5 3}$

$5^{\log_3 7} + 7^{\log_5 3} - 5^{\log_3 7} - 7^{\log_5 3} = 0$

QUESTION

Prove that $\frac{\log_2 24}{\log_{96} 2} - \frac{\log_2 192}{\log_{12} 2} = 3$ Tahol

QUESTION

Simplify & compute :

$$\frac{\log_5 250}{\log_{50} 5} - \frac{\log_5 10}{\log_{1250} 5}$$

Tanoz

QUESTION [JEE Advanced 2018]

The value of $\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}} \times \left(\sqrt{7}\right)^{\frac{1}{\log_4 7}}$ is

Tah 03

QUESTION [JEE Advanced 2013]

$$a = b \iff \log_c a = \log_c b \;,\; a, b \in R^+ \;,\; c > 0, c \neq 1$$

If $3^x = 4^{x-1}$, then $x =$

(A) $\dfrac{2\log_3 2}{2\log_3 2 - 1}$

(B) $\dfrac{2}{2 - \log_2 3}$

(C) $\dfrac{1}{1 - \log_4 3}$

(D) $\cancel{\dfrac{2\log_2 3}{2\log_2 3 - 1}}$

$$3^x = 4^{x-1}$$

$$\log_2 3^x = \log_2 4^{x-1}$$

$$x\log_2 3 = (x-1)\log_2 4$$

$$x\log_2 3 = (x-1)\cdot 2 = 2x - 2$$

$$2 = 2x - x\log_2 3$$

$$x(2 - \log_2 3) = 2$$

$$x = \frac{2}{2 - \log_2 3} \quad \text{(B)}$$

$$x = \frac{2}{2 - \log_2 3} = \frac{2}{2 - \frac{1}{\log_3 2}}$$

$$x = \frac{2\log_3 2}{2\log_3 2 - 1} \quad \text{(A)}$$

$$x = \frac{2}{2 - \log_2 3} = \frac{2}{2 - \log_{4^{1/2}} 3}$$

$$= \frac{2}{2 - 2\log_4 3} = \frac{2}{2(1 - \log_4 3)} = \frac{1}{1 - \log_4 3} \quad \text{(C)}$$

Logarithmic Equations

$a^{m+n} = a^m \cdot a^n$

$a^{\log_a b} = b$

$5^2 = 5 \times 5 = 25$

$5^{2+\log_5 x}$

$= 5^2 \cdot 5^{\log_5 x}$

$25 \cdot x$

$25x.$

* Base of logarithm should be same throughout

* Try simplify each term if possible. Ex: $5^{2+\log_5 x} = 5^2 \cdot 5^{\log_5 x} = 25x$

* If you take log on both sides of Eqn then take it to base already given in Question.

* verify All answers from original Eqn and check If any term becomes undefined or imaginary

$a^{\frac{1}{n}} = \sqrt[n]{a}$

$n \geq 2, n \in N$

$2 = \sqrt{4} = 4^{1/2}$

QUESTION

★★ASRQ★★

$\frac{\log_2(9-2^x)}{x-3} = 1$. Then number of solution is

(A) 3

(B) 2

(C) 1

(D) 0

$\log_2(9-2^x) = x-3$

$9-2^x = 2^{x-3}$

$9-2^x = 2^x \cdot 2^{-3}$

$9-2^x = \frac{2^x}{2^3}$ let $2^x = a$

$9-a = \frac{a}{8}$

$72-8a = a$

$9a = 72$

$a = 8$

$2^x = 8$

$2^x = 2^3 \Rightarrow x = 3$ (rejected)

QUESTION

$\log_{x-1} 4 = 1 + \log_2(x-1)$ then x is

(A) 3

(B) 2

(C) 5/4

(D) 4/5

$\log_{x-1} 2^2 = 1 + \log_2(x-1)$

$2\log_{x-1} 2 = 1 + \log_2(x-1)$

$\dfrac{2}{\log_2(x-1)} = 1 + \log_2(x-1) \longrightarrow$ let $\log_2(x-1) = t$

$\dfrac{2}{t} = 1 + t$

$t^2 + t - 2 = 0$

$(t+2)(t-1) = 0$

$t = -2, 1$

$\log_2(x-1) = -2, 1$

$x - 1 = 2^{-2}$ or $x - 1 = 2^1$

$x = 1 + \dfrac{1}{2^2} = 5/4,\ x = 3$

$x = 5/4, 3$

QUESTION

$a^x = N \Longleftrightarrow \log_a N = x$

$(x+1)^{\log_{10}(x+1)} = 100(x+1)$ then x is

(A) 99

(B) – 0.9

(C) 0.9

(D) 10

Taking log on both sides to base 10

$\log_{10}(x+1)^{\log_{10}(x+1)} = \log_{10}(100\cdot(x+1))$

$\log_{10}(x+1)\cdot\log_{10}(x+1) = \log_{10}100 + \log_{10}(x+1)$

let $\log_{10}(x+1) = t$

$t^2 = 2+t$

$t^2 - t - 2 = 0$

$(t-2)(t+1) = 0$

$t = 2, -1$

$\log_{10}(x+1) = 2$ or $\log_{10}(x+1) = -1$

$x+1 = 10^2$ or $x+1 = 10^{-1}$

$x = 99$, $x = \frac{1}{10} - 1 = -\frac{9}{10} = -0.9$

QUESTION

★★KCLS★★

$$a^{m+n+p} = a^m \cdot a^n \cdot a^p$$

$\log_5(\sqrt[x]{5} + 125) = \log_5 6 + 1 + \frac{1}{2x}$ then x is

(A) –1

(B) 1/2

(C) 1/16

(D) 1/4

$$\log_5(5^{1/x} + 125) = \log_5 6 + 1 + \frac{1}{2x}$$

$$5^{\frac{1}{x}} + 125 = 5^{\log_5 6 + 1 + \frac{1}{2x}} = 5^{\log_5 6} \cdot 5^1 \cdot 5^{\frac{1}{2x}}$$

$$5^{\frac{1}{x}} + 125 = 6 \cdot 5 \cdot 5^{\frac{1}{2x}}$$

$$5^{1/x} + 125 = 30 \cdot 5^{\frac{1}{2x}}$$

$$t^2 + 125 = 30t$$

$$t^2 - 30t + 125 = 0$$

$$t = 5, 25.$$

$$\cancel{5^{1/x} = t}$$

$$\cancel{\Downarrow}$$

$$\cancel{5^{1/2x} = t^2}$$

Gadho | Gadhiyaa aisaa naa karo!!

let $5^{1/2x} = t$

$$\left(5^{\frac{1}{2x}}\right)^2 = t^2$$

$$5^{1/x} = t^2$$

$$5^{\frac{1}{2x}} = 5^1, 5^2$$

$$\frac{1}{2x} = 1, 2$$

$$2x = 1, \frac{1}{2}$$

$$x = \frac{1}{2}, \frac{1}{4} \longrightarrow \cancel{B, D}$$

$x \notin N$

(wrong)

(No soln)

Reason

$\sqrt[x]{5}$, $x \in N$, $x \geq 2$

QUESTION

$\log_5(5^{1/x} + 125) = \log_5 6 + 1 + \frac{1}{2x}$ then x is

(A) -1

(B) $1/2$

(C) $1/16$

(D) $1/4$

$\sqrt[x]{5} = 5^{1/x}$, $x \in N$

$5^{1/x} = \sqrt[x]{5}$ only if $x \in N, x \geq 2$

$5^{3/4} \neq \sqrt[\frac{3}{4}]{5}$.

QUESTION

$5^{1+\log_4 x} + 5^{\log_{\frac{1}{4}} x - 1} = \frac{26}{5}$ then x is

(A) 1

(B) 1/16

(C) 1/2

(D) 16

$a^{m-n} = \frac{a^m}{a^n} = a^m \cdot a^{-n}$ ($a^{m+(-n)}$)

* $a^{-m} = \frac{1}{a^m}$
* $\log_2 4 - 1 = 2 - 1 = 1$
* $\log_3 (28-1) = \log_3 27 = 3$

Gadho/Gadhiyoo aisaa naa karo $5 \cdot 5^{1/x} = 25^{1/x}$

$5^1 \cdot 5^{\log_4 x} + 5^{\log_{\frac{1}{4}} x} \cdot 5^{-1} = \frac{26}{5}$

$5 \cdot 5^{\log_4 x} + \frac{5^{\log_{4^{-1}} x}}{5^1} = \frac{26}{5}$

$5 \cdot 5^{\log_4 x} + \frac{5^{-\log_4 x}}{5} = \frac{26}{5}$

$5 \cdot 5^{\log_4 x} + \frac{1}{5^{\log_4 x}} \cdot \frac{1}{5} = \frac{26}{5}$ let $5^{\log_4 x} = t$

$5t + \frac{1}{5t} = \frac{26}{5}$

$$25t^2+1=26t$$

$$25t^2-26t+1=0$$

$$25t^2-25t-t+1=0$$

$$(25t-1)(t-1)=0$$

$$t=\frac{1}{25},1$$

$$5^{\log_4 x}=5^{-2} \text{ or } 5^{\log_4 x}=5^0$$

$$\log_4 x=-2 \text{ or } \log_4 x=0$$

$$x=4^{-2}, \quad x=4^0$$

$$x=1/16, 1$$

QUESTION

★★★ ASRQ ★★★★

Find the square of the sum of the roots of the equation

$\log_3 x \cdot \log_4 x \cdot \log_5 x = \log_3 x \cdot \log_4 x + \log_4 x \cdot \log_5 x + \log_5 x \cdot \log_3 x.$

Case ① If $\log_3 x \cdot \log_4 x \cdot \log_5 x \neq 0$

$1 = \frac{1}{\log_5 x} + \frac{1}{\log_3 x} + \frac{1}{\log_4 x}$

$1 = \log_x 5 + \log_x 3 + \log_x 4$

$1 = \log_x (5 \times 3 \times 4)$

$1 = \log_x 60$

$x^1 = 60$

Case (II) If $\log_3 x \cdot \log_4 x \cdot \log_5 x = 0$

i.e if $x = 1$

LHS = 0 = RHS

$\Downarrow$

$x = 1$ is also a soln

Ans: $(60+1)^2 = 3721$

Gadho | Gadhiyoo aisaa naa karo

QUESTION

★★KCLS★★

Solve for x : $\log^2(4-x)+\log(4-x)\cdot\log\left(x+\frac{1}{2}\right)-2\log^2\left(x+\frac{1}{2}\right)=0$.

let $\log(4-x)=a$

$\log\left(x+\frac{1}{2}\right)=b$

$a^2+ab-2b^2=0$

$a^2+2ab-ab-2b^2=0$

$a(a+2b)-b(a+2b)=0$

$(a+2b)(a-b)=0$

$a=b,-2b.$

$\log(4-x)=\log(x+1/2)$ or $\log(4-x)=-2\log\left(x+\frac{1}{2}\right)$

$4-x=x+1/2$

$x=7/4$

$\log(4-x)=\log\left(\frac{2x+1}{2}\right)^{-2}$

$4-x=\frac{4}{(2x+1)^2}$

$A\cdot f^{2n}(x)+B\cdot f^{n}(x)\cdot g^{m}(x)+C\cdot g^{2m}(x)=0$

Try splitting middle term as in quad

Ex: $x^4+5x^2y^2+6y^4$

$x^4+3x^2y^2+2x^2y^2+6y^4$

$(x^2+2y^2)(x^2+3y^2)$

$$(4-x)(2x+1)^2 = 4$$

$$(4-x)(4x^2+1+4x) = 4$$

$$16x^2 + \cancel{4} + 16x - 4x^3 - x - 4x^2 = \cancel{4}$$

$$-4x^3 + 12x^2 + 15x = 0$$

$$-x(4x^2 - 12x - 15) = 0$$

$$x = 0, \quad 4x^2 - 12x - 15 = 0$$

$$x = \frac{12 \pm \sqrt{144+240}}{8}$$

$$x = \frac{12 \pm \sqrt{4 \times 16 \times 6}}{8}$$

$$x = \frac{3 \pm 2\sqrt{6}}{2}$$

384

96.

$$x + \frac{1}{2} = \frac{3-2\sqrt{6}}{2} + \frac{1}{2}$$

$$= \frac{4-2\sqrt{6}}{2} = -ve$$

$$x = 0, 7/4, \frac{3+2\sqrt{6}}{2}, \cancel{\frac{3-2\sqrt{6}}{2}}$$

$$x + 1/2 < 0$$

QUESTION

Tanoy

Indicate all correct alternatives, where base of the log is 2.

The equation $x^{\frac{3}{4}(\log_2 x)^2+\log_2 x-\frac{5}{4}} = \sqrt{2}$ has :

A) At least one real solution

B) Exactly 3 real solutions

C) Exactly one irrational solution

D) Imaginary roots

QUESTION

Tah 05

The equation $x^{\left[(\log_3 x)^2-\frac{9}{2}\log_3 x+5\right]}=3\sqrt{3}$ has

(A) Exactly 3 real solutions

(B) At least one real solution

(C) Exactly one irrational solution

(D) Complex roots

Saari Class Illustrations
Retry karni Hai

Home Challenge-03

Positive integers a and b satisfy the condition $\log_2\left[\log_{2^a}\left(\log_{2^b}(2^{1000})\right)\right] = 0$. Then the possible values of $a + b$ is/are:

(A) 501

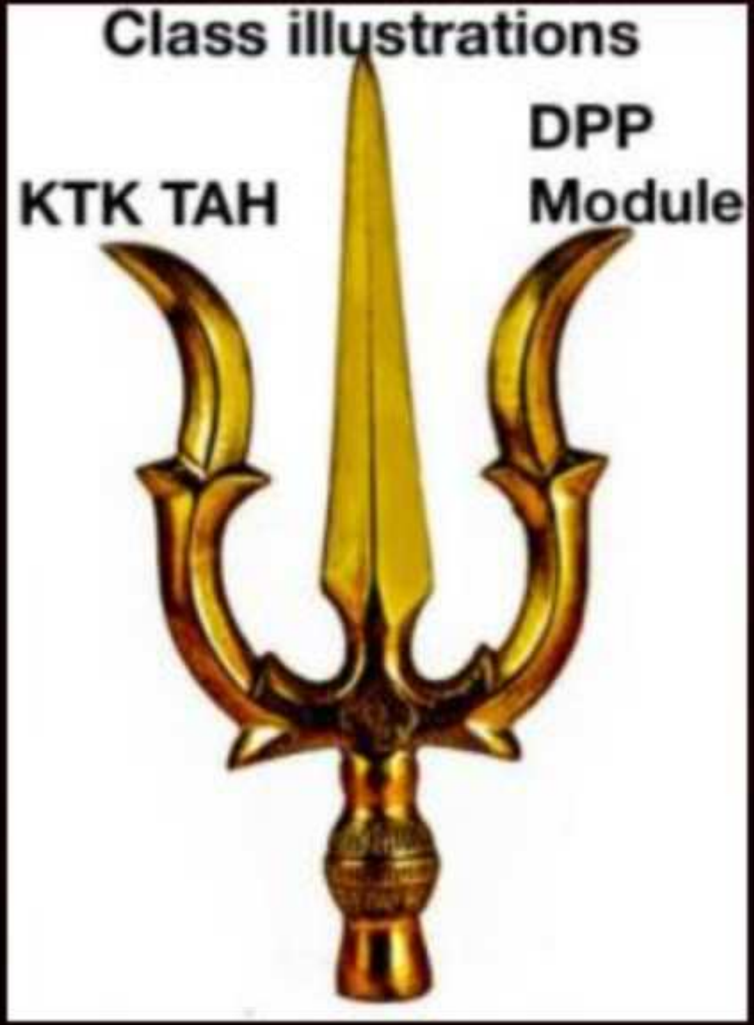
(B) 252

(C) 128

(D) 66

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with Good Rank



QUESTION

(KTK 1)

Solve the following inequalities:

(1) $\frac{x}{x-5} > \frac{1}{2}$

(2) $\frac{x-5}{x-9} \leq 0$

(3) $x \leq 3 - \frac{1}{x-1}$

(4) $\frac{1}{x} \leq 1$

(5) $\frac{5x}{3x-1} \leq 0$

Ans. (1) $x \in (-\infty, -5) \cup (5, \infty)$, (2) $x \in (-\infty, 0) \cup [1, \infty)$, (3) $x \in [5, 9)$, (4) $x \in \left[0, \frac{1}{3}\right)$, (5) $x \in (-\infty, 1) \cup \{2\}$

QUESTION

(KTK 2)

Complete solution set of inequality $\frac{(x+2)(x+3)}{(x-2)(x-3)} \leq 1$ is

(A) $(-\infty, 0]$

(B) $(-\infty, 0] \cup (2, 3)$

(C) $[2, 3]$

(D) $(-\infty, 2) \cup (3, \infty)$

Ans. B

QUESTION

(KTK 3)

Find sum of all integral values of x satisfying $\frac{x^2-5x+6}{x^2-x-12} \leq 0$.

Ans. 3

QUESTION

(KTK 4)

Which of the following does not hold true for the expression
$E = \sqrt{x^2 - 2x + 1} - \sqrt{x^2 + 2x + 1}$?

(A) $E = 2$ if $x \leq -1$

(B) $E = -2x$ if $-1 < x < 1$

(C) $E = -2$ if $x \geq 1$

(D) $E = -2$ for all x

Ans. D

QUESTION

KTK 07

If $x \in [-5, 7]$, then number of integral values of x satisfying $\frac{2x+3}{x^2+x-12} < \frac{1}{2}$ is

(A) 5

(B) 6

(C) 7

(D) 8

Ans. C

QUESTION

KTK 08

Solution set of the inequality $x + 1 \leq \frac{6}{x}$ is

(A) $(0, 2]$

(B) $[-3, 2)$

(C) $(-\infty, -3] \cup (0, 2]$

(D) $[-3, 0) \cup (2, \infty)$

Ans. C

QUESTION

KTK 09

The set of all values of x for which $\frac{(x+1)(x-3)^2(x-5)(x-4)^3(x-2)}{x} < 0$ is

(A) $(-\infty,-1) \cup (0,2) \cup (4,5)$

(B) $(-1,0) \cup (2,4) \cup (5,\infty)$

(C) $(-1,0) \cup (2,3) \cup (4,5)$

(D) $(-\infty,-1) \cup (0,2) \cup [3,5)$

Ans. A

QUESTION

KTK 10

Which of the following sets does not satisfy the inequality $\frac{1}{x-2} + \frac{1}{x-1} \geq \frac{1}{x}$?

A $(-\sqrt{2}, 0)$

B $(1, \sqrt{2})$

C $(2, \infty)$

D $(0, 1)$

Ans. D

Solution to Previous TAH

QUESTION

Solve: $(x^2 - x - 1)(x^2 - x - 7) < -5$

Ans. $x \in (-2, -1) \cup (2, 3)$

$$(x^2-x-1)(x^2-x-7) < -5$$

Tah-01

Sol

Let, $x^2 - x = t$

$$(t-1)(t-7) < -5$$

$$t^2 - 7t - t + 7 < -5$$

$$t^2 - 8t + 7 < -5$$

$$t^2 - 8t + 12 < 0$$

$$t^2 - 6t - 2t + 12 < 0$$

$$t(t-6) - 2(t-6) < 0$$

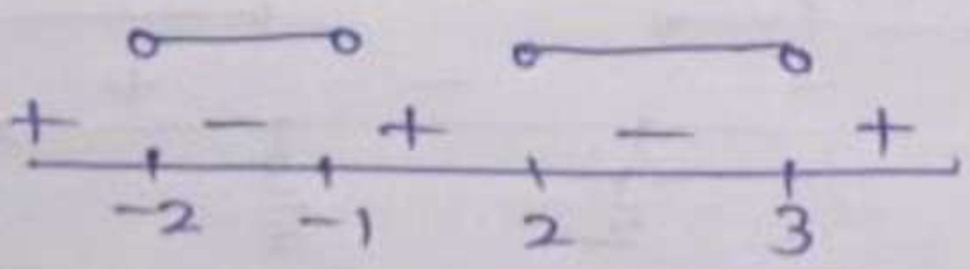
$$(t-2)(t-6) < 0$$

$$(x^2-x-2)(x^2-x-6) < 0$$

$$(x^2-2x+x-2)(x^2-3x+2x-6) > 0$$

$$x(x-2)+1(x-2) \qquad x(x-3)+2(x-3)$$

$$(x+1)(x-2)(x+2)(x-3) < 0$$



$$x \in (-2,-1) \cup (2,3)$$

• Q (TAH) - 1: Solve: $(x^2-x-1)(x^2-x-7)<-5$.

TAH 1

Soln: let $x^2-x=t$.

$$(t-1)(t-7)<-5$$

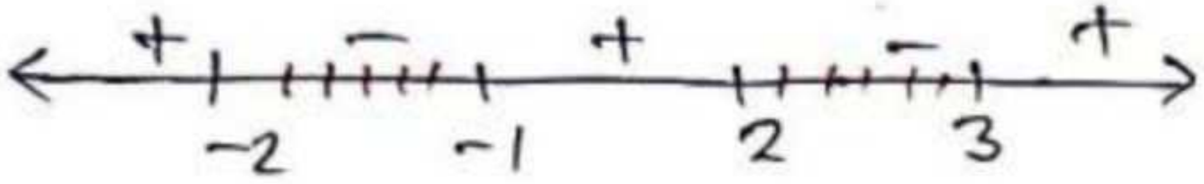
or, $t^2-7t-t+7<-5$

or, $t^2-8t+12<0$

or, $(t-6)(t-2)<0$

or $(x^2-x-6)(x^2-x-2)<0$

or $(x-3)(x+2)(x-2)(x+1)<0$



$\therefore\ x\in(-2,-1)\cup(2,3)$

QUESTION

If $\frac{x^3(x-1)^2(x+4)}{(x+1)(x-3)} \geq 0$, then $x \in$

(A) $(-\infty, -4]$

(B) $(-1, 0]$

(C) $(3, \infty)$

(D) $\{1\}$

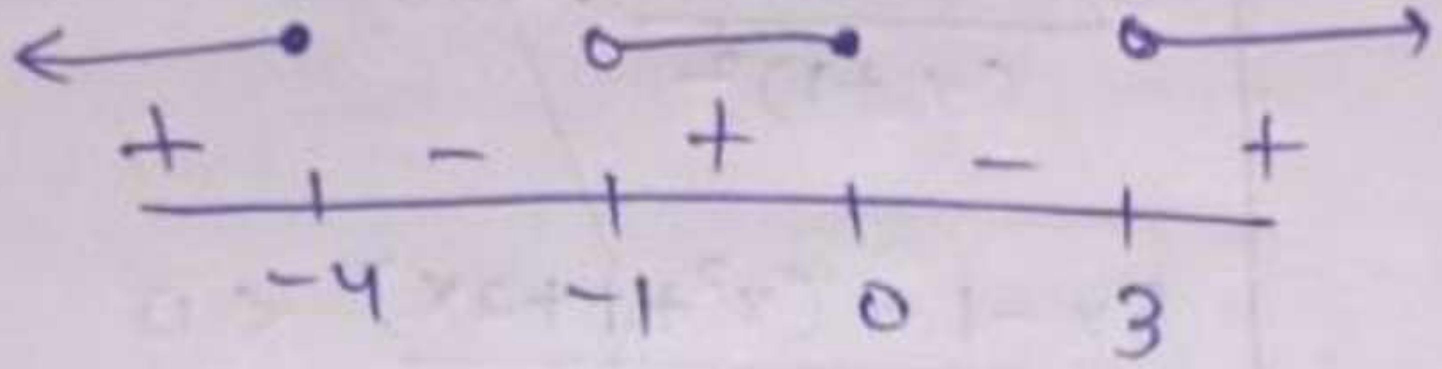
Ans. A, B, C, D

13. If $\dfrac{x^3(x-1)^2(x+4)}{(x+1)(x-3)} \geq 0$ then $x \in$

Tah-02

$x = 1$ is also possible

$$\frac{x^3(x+4)}{(x+1)(x-3)} \geq 0$$



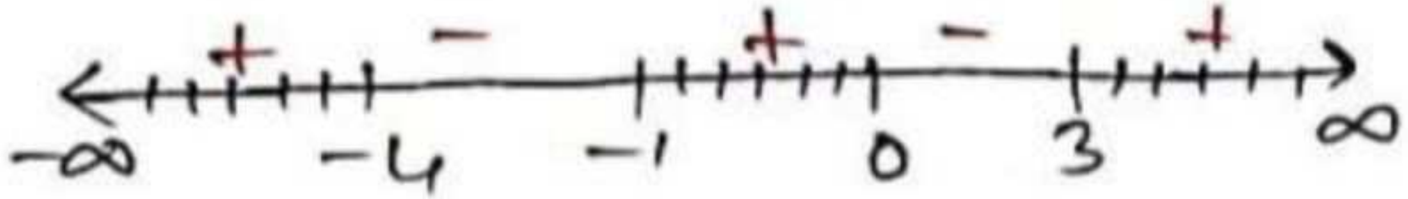
$$x \in (-\infty, -4] \cup (-1, 0] \cup (3, \infty) \cup \{1\}$$

TAH-2: $\dfrac{x^3(x-1)^2(x+4)}{(x+1)(x-3)} \geq 0 \; ; x \in ?$

TAH 2

Sol^n: $\dfrac{x^3(x-1)^2(x+4)}{(x+1)(x-3)} \geq 0.$

$\Rightarrow \dfrac{x(x+4)}{(x+1)(x-3)} \geq 0$; $x=1$ is also possible.



$\therefore x \in (-\infty, -4] \cup (-1, 0] \cup (3, \infty) \cup \{1\}$

QUESTION

TAH 03

1. Solve $\frac{x(3-4x)(x+1)}{(2x-5)} < 0$ [Ans. $x \in (-\infty, -1) \cup (0, 3/4) \cup (5/2, \infty)$]

2. Solve $\frac{(2x+3)(4-3x)^3(x-4)}{(x-2)^2x^5} \leq 0$ [Ans. $x \in (-\infty, -3/2) \cup (0, 4/3] \cup [4, \infty)$]

3. Solve $\frac{(x-3)(x+5)(x-7)}{|x-4|(x+6)} \leq 0$ [Ans. $x \in (-6, -5] \cup [3, 4) \cup (4, 7]$]

4. Solve $\frac{5x+1}{(x+1)^2} < 1$ [Ans. $x < 0$ or $x > 3, x \neq -1$]

5. Solve $\frac{x^4}{(x-2)^2} > 0$ [Ans. $x \in R - \{0, 2\}$]

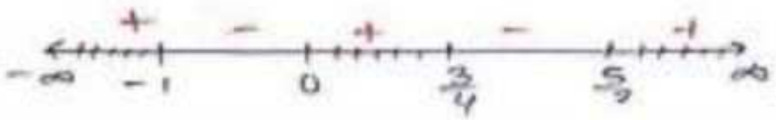
6. Solve $\frac{6x^2-5x-3}{x^2-2x+6} \leq 4$ [Ans. $-\frac{9}{2} \leq x \leq 3$]

7. Solve $\frac{(x+2)(x^2-2x+1)}{-4+3x-x^2} \geq 0$ [Ans. $x \in (-\infty, -2] \cup \{1\}$]

TA II-3: Solve the inequalities

Soln:- (i) $\dfrac{x(3-4x)(x+1)}{(2x-5)} < 0$

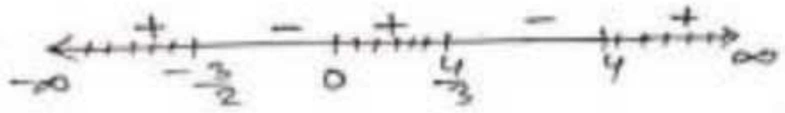
$\Rightarrow \dfrac{x(4x-3)(x+1)}{(2x-5)} > 0$



$\therefore\ x \in (-\infty,-1) \cup (0, \tfrac{3}{4}) \cup (\tfrac{5}{2}, \infty)$ Ans.

(ii) $\dfrac{(2x+3)\,(4-3x)^3\,(x-4)}{(x-2)^2 \cdot x^5} \leq 0$

$\Rightarrow \dfrac{(2x+3)\,(3x-4)\,(x-4)}{x} \geq 0$; $x \neq 2$



$x \in (-\infty, -\tfrac{3}{2}] \cup (0, \tfrac{4}{3}] \cup [4, \infty)$ Ans.

(iii) $\dfrac{5x+1}{(x+1)^2} < 1$

$\Rightarrow \dfrac{5x+1-x^2-2x-1}{(x+1)^2} < 0$

$\Rightarrow \dfrac{-x^2+3x}{(x+1)^2} < 0$

$\Rightarrow \dfrac{x(x-3)}{(x+1)^2} > 0$

$\Rightarrow x(x-3) > 0$; $x \neq -1$

$\therefore\ x \in (-\infty,0) \cup (3,\infty) - \{-1\}$

$\Rightarrow x \in (-\infty,-1) \cup (-1,0) \cup (3,\infty)$ Ans.

TAH 3 by Reed from WB

TAH-3:

(v) $\dfrac{x^4}{(x-2)^2} > 0$

$[x \neq 0]$

$\Rightarrow \dfrac{x^4}{(x-2)^2}$ $x^4 \rightarrow$ always +ve for $\forall x \in R-\{0\}$

$(x-2)^2$ always +ve for $\forall x \in R-\{2\}$ $\Rightarrow x \in R-\{2,0\}$ Ans

$[x \neq 2]$

(vi) $\dfrac{6x^2-5x-3}{x^2-2x+6} \leq 4$

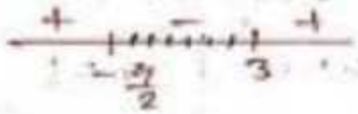
$\Rightarrow \dfrac{6x^2-5x-3-4(x^2-2x+6)}{x^2-2x+6} \leq 0$

$x^2-2x+6 \rightarrow D<0,\ a>0 \Rightarrow$ always +ve

$\Rightarrow 2x^2+3x-27 \leq 0$

$\Rightarrow 2x^2+9x-6x-27 \leq 0$

$\Rightarrow (2x+9)(x-3) \leq 0.$



$\Downarrow$

$x \in [-\tfrac{9}{2}, 3].$

TAH 3(part2) By Reed from WB

(viii) $\dfrac{(x+2)(x^2-2x+1)}{-4+3x-x^2} \geq 0$

$\Rightarrow \dfrac{(x+2)\,(x-1)^2}{(x^2-3x+4)} \leq 0$

$x^2-3x+4 \rightarrow D<0,\ a>0$ always +ve.

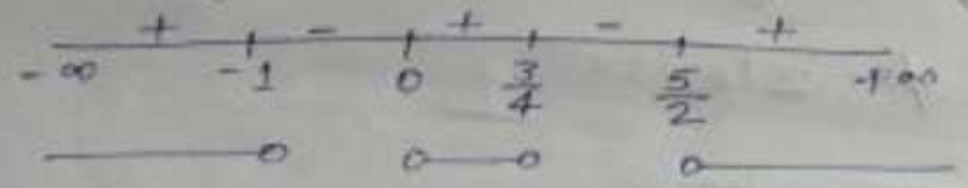
$\Rightarrow \dfrac{(x+2)}{1} \leq 0$ & $x = 1$ is also possible.

$\Rightarrow \boxed{x \leq -2}$. $\quad x \in (-\infty,-2] \cup \{1\}$

Tah-03

(i) $\dfrac{x(3-4x)(x+1)}{(2x-5)} < 0$

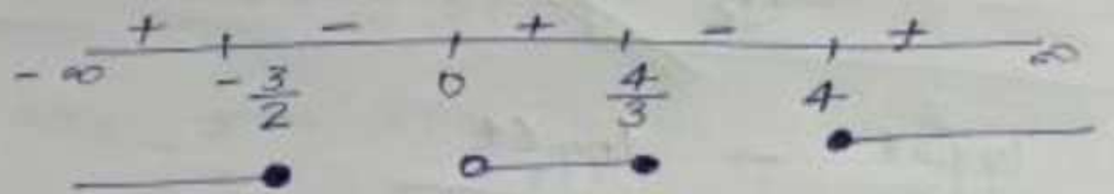
$\Rightarrow \dfrac{x(4x-3)(x+1)}{(2x-5)} > 0 \rightarrow +ve$



$x \in (-\infty,-1) \cup (0, \frac{3}{4}) (\frac{5}{2}, \infty)$ Ans.

(ii) $\dfrac{(2x+3)(4-3x)^3 (x-4)}{(x-2)^2 \; x^5} \leq 0$

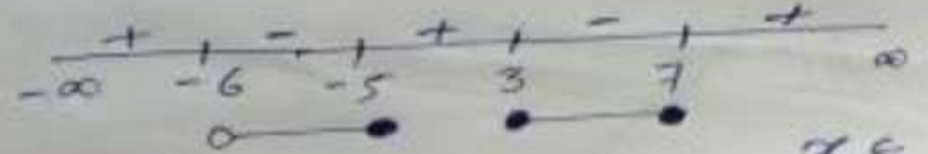
$\Rightarrow \dfrac{(2x+3)(3x-4)^3 (x-4)}{x^5} \geq 0 \quad x \neq 2.$ (+ve)



$x \in (-\infty, \frac{-3}{2}] \cup (0, \frac{4}{3}] \cup [4, \infty)$ Ans

(iii) $\dfrac{(x-3)(x+5)(x-7)}{|x-4| \; (x+6)} \leq 0$

$\Rightarrow \dfrac{(x-3)(x+5)(x-7)}{(x+6)} \leq 0 \quad x \neq 4$ (-ve)



$x \in (-6,-5] \cup [3,7] - \{4\}$

(4) $\dfrac{5x+1}{(x+1)^2} < 1$

TAH-03 (04)

$\Rightarrow \dfrac{5x+1}{(x+1)^2} - 1 < 0$

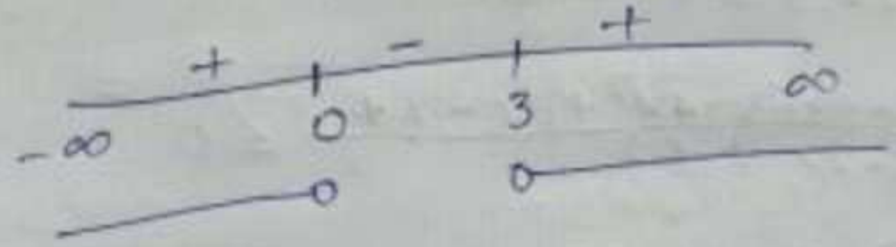
$\Rightarrow \dfrac{5x+1-(x+1)^2}{(x+1)^2} < 0$

$\Rightarrow \dfrac{5x+1-x^2-2x-1}{(x+1)^2} < 0$

$\Rightarrow \dfrac{-x^2+3x}{(x+1)^2} < 0$

$\Rightarrow \dfrac{-x(x-3)}{(x+1)^2} < 0$

$\Rightarrow \dfrac{x(x-3)}{(x+1)^2} > 0 \Rightarrow x(x-3) > 0 \quad x \neq 1$ (+ve.)



$x \in (-\infty, 0) \cup (3, \infty)$ Ans

(5)

(5) $\dfrac{x^4}{(x-2)^2} > 0 \rightarrow +ve$

TAH-03
(05)

$+ \quad + \quad +$

$-\infty \quad 0 \quad 2 \quad \infty$

$x \neq 0, 2$

$x \in R - \{2\}\{0\}$ Ans

(96) $\frac{6x^2-5x-3}{x^2-2x+6}-4 \le 0$

$\Rightarrow \frac{6x^2-5x-3-4x^2+8x-24}{(x^2-2x+6)} \le 0$

$\Rightarrow \frac{2x^2+3x-27}{x^2-2x+6} \le 0$

$\Rightarrow \frac{2x^2+9x-6x-27}{(x^2-2x+6)} \le 0$

$\Rightarrow \frac{(2x+9)(x-3)}{(x^2-2x+6)} \le 0$

$\to$ $D<0$
$a>0$
x^2-2x+6 always +ve

$-\infty$ + $-\frac{9}{2}$ − 3 + ∞

$x \in \left[\frac{-9}{2}, 3\right]$ Ans

Tah-03
(6)

(7) $\dfrac{(x+2)(x^2-2x+1)}{-4+3x-x^2} \geq 0$

TAH-03 (07)

$\Rightarrow \dfrac{(x+2)(x^2-2x+1)}{x^2-3x+4} \leq 0$

$D<0$

$a>0$

$\Rightarrow \dfrac{(x+2)(x-1)(x-1)}{\cancel{(x-4)}} \leq 0$

$-\infty \quad - \quad -2 \quad + \quad \infty$

$x \neq 1$

$x \in (-\infty, -2]$

QUESTION

Find all values of x for which the following equalities hold true.

(i) $\log_2 x^2 = 1$

(ii) $\log_3 x = \log_3(2 - x)$

(iii) $\log_4 x^2 = \log_4 x$

(iv) $\log_{1/2}(2x + 1) = \log_{1/2}(x + 1)$

(v) $\log_{1/3}(x^2 + 8) = -2$

TAH-4: Find all values of x for which the following equalities hold true:

(i) $\log_2 x^2 = 1$ (ii) $\log_3 x = \log_3(2-x)$

(iii) $\log_4 x^2 = \log_4 x$ (iv) $\log_{1/2}(2x+1) = \log_{1/2}(x+1)$

(v) $\log_{1/3}(x^2+8) = -2$

Soln: (i) $\log_2 x^2 = 1$

$\Rightarrow x^2 = 2^1$

$\Rightarrow \boxed{x = \pm\sqrt{2}}$

$\therefore$ Equality holds at $x = \sqrt{2}, -\sqrt{2}$

(ii) $\log_3 x = \log_3(2-x)$

$\Rightarrow x = 2-x$

$\Rightarrow 2x = 2$

$\Rightarrow \boxed{x = 1}$

$\therefore$ Equality holds at $x = 1$.

(iii) $\log_4 x^2 = \log_4 x$

$\Rightarrow x^2 = x$

$\Rightarrow x(x-1) = 0$

$x = 0$ N.P. | $x = 1$ (Fine)

$\therefore$ Equality holds at $x = 1$.

(iv) $\log_{\frac{1}{2}}(2x+1) = \log_{\frac{1}{2}}(x+1)$

$\Rightarrow 2x+1 = x+1$

$\Rightarrow \boxed{x = 0}$ (valid)

$\therefore$ Equality holds at $x = 0$.

(v) $\log_{\frac{1}{3}}(x^2+8) = -2$

$\Rightarrow x^2+8 = \left(\frac{1}{3}\right)^{-2} = 9$

$\Rightarrow x^2 = 1$

$\Rightarrow \boxed{x = \pm 1}$

$\therefore$ Equality holds at $x = 1, -1$.

TAH 4
BY REED
FROM WB

THANK YOU